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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number FIVE (Due: Sat. at 1pm November 10)

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QUESTION 1. (i) Let $G = (Z, +)$. Prove that every proper subgroup H (i.e., $H \neq \{0\}$ and $H \neq Z$) of G is of the form $kZ = \{ka | a \in Z\}$ for some positive integer $k > 1$.

(ii) Let M be a proper subgroup of $(Z, +)$. Prove that M is a group-isomorphic to Z.

(iii) We know (it is easy to check) that $N = 12Z \cap 16Z$ is a subgroup of Z. Hence $N = kZ$ by (i). Find k. In general, $L = mZ \cap nZ$ (m, n are some positive integers greater than 1) is a proper subgroup of Z. Hence $L = kZ$. What should k be?

(iv) Give me an example of an infinite group D that has a normal subgroup H such that D/H is a finite group.

(v) Give me an example of a group D that has a normal subgroup H such that D/H is infinite but each element in D/H is of finite order.

(vi) Let F be a non-trivial group homomorphism from $(Z, +)$ into (Q^*, X). Prove that either $Ker(F) = \{0\}$ or $Ker(F) = 2Z$.

(vii) Prove that (Q^*, X) is not a cyclic group.

(viii) Is $Z_2 \oplus Z_4$ isomorphic to Z_8? If yes, then construct a group isomorphism between them. If not, then explain

(ix) Construct a non-trivial group homomorphism from A_4 into Z_{12}.

(x) Construct a non-trivial group homomorphism from Z_6 into Z_{10}.

(xi) Prove that $Z_2 \oplus Z_2$ is a group-isomorphic to $U(8)$.

Faculty information

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